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TIME-DOMAIN QUASI-LINEAR IDENTIFICATION OF NONLINEAR DYNAMIC SYSTEMS

83-0811

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Abstract

A time-domain linear modal identification technique is applied to identify some highly nonlinear dynamic systems. The modal concept is used to identify such nonlinear systems with the understanding that the resulting modes are only a mathematical representation of the series solution of the nonlinear system under consideration. Naturally these identified modal parameters are not unique, for nonlinear systems, since they are functions of the systems' amplitudes and hence referred to as quasi-linear. The approach presented in this paper can be useful in predicting signs of nonlinearities when linearity is assumed. It can also be used to analyze and understand types of nonlinearities of nonlinear systems through successive identifications at different levels of responses.

Nomenclature

f	frequency in Hz
F_i	force in restoring force element i
M	mass
m	number of degrees of freedom of the identification math model
n	number of harmonics
$\{n(t)\}$	measurements noise vector
$\{N_i\}$	modal vector for noise representation
p	number of degrees of freedom of system under identification
q	number of degrees of freedom allowed for measurements noise
r_i	the ith characteristic root for noise representation
$\{x(t)\}$	linear system response vector
$\{y(t)\}$	nonlinear system response vector
$\{z(t)\}$	displacement in restoring force element
α_i	the ith characteristic root for harmonics
Γ	the ith modal vector of harmonics
ζ_i	the ith damping factor (%)
θ	angular displacement
$\{\phi_i\}$	the ith vector in the systems response matrix $[\Phi]$
$\{\psi_i\}$	the ith linear (or equivalent linear) modal vector
ITD	Ibrahim Time Domain modal identification technique
NMO	number of modes allowed in the identification math model
SDOF	single degree-of-freedom system
TDOF	two degrees-of-freedom system

Introduction

With the increasing complexity of modern aerospace and non-aerospace structures, accurate dynamic identification has become a necessity. Dynamic identification is usually carried out through identifying the structure's modal parameters. These modal parameters are required for modeling, responses and loads prediction, stability analysis and control system design.

The demand for more sophisticated dynamic identification techniques, to match the stringent accuracy requirements, for dynamic design and performance analysis, has resulted in numerous research efforts in this area during the last two decades.

Presently, for dynamic identification, a structural dynamicist has a choice between frequency domain-techniques[1,2] and time-domain techniques.[3-11] Although quite different and their merits are still and will be debatable for a while, the two approaches so far have been dealing with only linear systems.

There have been few efforts for the dynamic identification of nonlinear dynamic systems.[12-19] Unfortunately these efforts are limited to lumped parameter systems and are still academic and far away from being applicable to real structures that possess some unknown forms of nonlinearity as well as unknown number of degrees of freedom.

The assumption of linearity in dynamic identification has been found to be a reasonable assumption for many applications. This is true where amplitudes of vibration are small or when in general the levels of nonlinearities are small and can be ignored. On the other hand, some applications require serious considerations for their high levels of nonlinearities where assuming linearity can be highly erroneous. An example of such applications is the case of large amplitude responses of panels subjected to acoustic and mechanical excitation.[20-26]

The efforts presented in this paper are first to study the applicability of a linear modal identification technique to nonlinear systems. The term quasi-linear used in this paper is meant to perform the identification at one certain level of excitation or response. Although the modal approach may be nonexistent for a nonlinear system as a whole, the modal concept will be used here and it is understood that the identified modal parameters will be function of the level of response of the system.

The second purpose of this paper is to identify the type of nonlinearities in the system. This can be attained by identifying the quasi-linear modal parameters of the system at different levels of responses and study the changes in these modal parameters.

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The linear modal identification method selected here for the quasi-linear identification of nonlinear systems is a time-domain technique referred to as the Ibrahim Time Domain (ITD) technique,[3-11] Appendix A.

It is to be noted here that although the examples used in this paper are lumped parameter systems, the method is applicable as well to distributed parameter systems. The identification technique is not dependent on the number of degrees of freedom of the system under consideration. The identification model allows any larger number of degrees of freedom such that all modal information in the responses can be identified.

Theory: Linearized Identification Model

For linear systems, the ITD technique is based on that the free-decay responses of a structure $\{x(t)\}$ are linear combinations of the excited modes:

$$\{x(t)\} = \sum_{i=1}^{2p} \{\psi_i\} e^{\lambda_i t} + \{n(t)\} \qquad (1)$$

where $\{\psi_i\}$ is the ith modal vector, λ_i is the ith characteristic root, p is the number of modes excited in the responses ($2p$ complex conjugate modes), and $n(t)$ is measurements noise.

The linear ITD technique also uses the concept of an oversized identification model[11] to reduce effects of measurement noise on the identified parameters. It was shown that allowing more degrees of freedom in the identification model improves accuracy of identification since the extra degrees of freedom act as outlets for the noise and equation (1) becomes

$$\{x(t)\} = \sum_{i=1}^{2p} \{\psi_i\} e^{\lambda_i t} + \sum_{k=2p+1}^{2m} \{N_k\} e^{r_k t} \qquad (2)$$

where m is greater than p.

This same concept can be used to develop a linearized identification model for nonlinear responses. The free-decay responses of a p degrees-of-freedom nonlinear system can be expressed as:

$$\{y(t)\} = \sum_{i=1}^{2p} \{\psi_i\} e^{\lambda_i t} + \sum_{k=1}^{\infty} \{\Gamma_k\} e^{\alpha_k t} + \{n(t)\} \qquad (3)$$

where in this case the first set of modes represents the fundamental solutions and the second set represents the harmonics.

If only a finite number of m degrees-of-freedom ($m > p$) are allowed in the identification model then equation (3) becomes

$$\{y(t)\} = \sum_{i=1}^{2p} \{\psi_i\} e^{\lambda_i t} + \sum_{k=1}^{2n} \{\Gamma_k\} e^{\alpha_k t} + \sum_{\ell=1}^{2q} \{N_\ell\} e^{r_\ell t} \qquad (4)$$

where

$$p + n + q = m$$

With the understanding that usually the amplitudes of higher harmonics get smaller for higher orders, the number of high harmonics to be identified will be dependent on identification accuracy and levels of noises in the responses.

Applications

To test the validity and applicability of the preceding theory, the proposed approach is applied to three different nonlinear systems. These systems were selected to represent single-degree-of-freedom systems with and without damping and a damped two-degrees-of-freedom system. The nonlinear terms are restricted to the stiffness terms while the damping terms were kept linear. For nonlinear stiffness, soft and hard springs are represented. High nonlinearities were achieved through having the nonlinear term coefficient larger than that of the linear term and or having large amplitudes of responses.

The simulated responses of these systems were obtained by numerically integrating the nonlinear differential equations with some specified initial displacements and zero initial velocities. A fourth order Runge-Kutta with variable step method was used for the numerical integration.

Simulated Systems

Three systems are simulated and identified using the preceding theory. These systems are:

1. A Simple Pendulum:

The nonlinear differential equation of motion of the simple pendulum, Fig. 1, is:

$$\ddot{\theta} + 4\pi^2 \sin\theta = 0 \qquad (5a)$$

$$\ddot{\theta} + 0.04\pi\dot{\theta} + 4\pi^2 \sin\theta = 0 \qquad (5b)$$

where equation (5a) represents the undamped case with a linear natural frequency of 1.0 Hz and equation (5b) has a 1.0% equivalent damping factor.

Responses were simulated for initial amplitudes θ_o of $\pi/6$, $\pi/3$, $\pi/2$, and $2\pi/3$.

2. A Mass-Spring System:

A single-degree-of-freedom system, Fig. 2, was designed to have hardening spring with an equivalent linear frequency of 1.0 Hz and equivalent linear damping of 1.0%. The governing equations of motion of such a system for undamped and damped cases are:

$$\ddot{y} + 4\pi^2(y + 2.0y^3) = 0 \qquad (6a)$$

$$\ddot{y} + 0.04\pi\dot{y} + 4\pi^2(y + 2.0y^3) = 0 \qquad (6b)$$

Responses were simulated for initial amplitudes of 0.5, 1.0, 1.5 and 2.0 units.

3. Two-Degrees-of-Freedom System:

To simulate a multi-degree-of-freedom nonlinear system with damping, a nonlinear system with two masses, three linear viscous dampers and three nonlinear springs, Fig. 3, is analyzed. The two springs were selected with cubic nonlinearity representing hardening springs with the coefficients of the nonlinear term being 50 and 100% of that of the linear term. The equations of motion of such system are:

$$\begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} + \begin{bmatrix} 0.06\pi & -0.02\pi \\ -0.02\pi & 0.06\pi \end{bmatrix} \begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{Bmatrix} + \begin{bmatrix} 10\pi^2 & -6\pi^2 \\ -6\pi^2 & 10\pi^2 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}$$

$$+ \begin{bmatrix} 7\pi^2 & -3\pi^2 \\ -3\pi^2 & 7\pi^2 \end{bmatrix} \begin{Bmatrix} y_1^3 \\ y_2^3 \end{Bmatrix} = \{0\} \qquad (7)$$

Free responses due to initial displacements were obtained by numerically integrating equation (7). Two sets of initial conditions were used. The first was to represent small amplitudes {0.3, 0.1} and the second set was to simulate larger amplitudes {3.0, 1.0}.

Identification

The numerically integrated responses were sampled at the rate of 50 Hz for the simple pendulum and the spring mass system and 100 Hz for the two-degrees-of-freedom system. Four seconds (200 samples) were used for the identification of both the SDOF and the TDOF systems. The number of modes allowed in the identification program was changed from 1 to 6 for the SDOF systems and from 1 to 12 for the TDOF system. Two samples were used to create the pseudo stations for the SDOF systems and six samples for the TDOF system. The parameter for delaying $[\hat{\phi}]$ from $[\phi]$ was taken as two samples for the SDOF system and four for the TDOF. This means an aliasing frequency of 12.5 Hz for all cases. Unaccounting for the errors arising from the numerical integration, cases with 0.0% and 1.0% noise/signal ratios were considered.

Discussions

1. The Undamped Case (SDOF)

For the simple pendulum and single mass-spring system, the ITD was able to identify very accurately the fundamental frequency and also harmonics up to the ninth harmonic. Tables 1 and 2 list the identified harmonics for the two cases and Figs. 4 and 5 show these identified frequencies. Table 3 lists identified damping factors for the undamped simple pendulum.

Tables 4 and 5 list the level of contribution of each harmonic showing the extremely small levels of contribution of higher harmonics.

Figures 6 and 7 show the identified fundamental frequencies, as functions of the initial amplitude, and their relation to the theoretical frequency of the nonlinear system. The same results are also shown in Tables 6 and 7.

As expected more harmonics are identifiable for higher levels of nonlinearities. This also is evident from determinant plots shown in Figs. 8 and 9. For smaller initial amplitudes, smaller nonlinearities, the determinants decreased at a faster rate indicating a lesser number of harmonics.

2. The Damped Case (SDOF)

The identification technique identified a strong fundamental frequency and very weak signs of a third harmonic. Without noise added to the responses, early signs of singularities were noticed when the number of degrees of freedom was increased beyond two, an indication of extremely small nonidentifiable higher harmonics.

For noise-free data larger number of degrees-of-freedom in the identification model revealed the changing frequency due to the decreasing amplitude of response due to damping. The accuracy of the identification program detected the change in frequency between the measurement response and the pseudo measurements. This phenomenon was not found when a small amount of noise, 1.0%, was added to the response. This can also be avoided by using shorter time records for identification.

3. The Two-Degrees-of-Freedom System

Equations (7) were integrated with two sets of initial conditions (0.3, 0.1) and (3.0, 1.0). These are only the initial conditions; responses in the second set had maximum displacements of 3.0 for both measurements.

Identification results showed no more than two modes for the small displacement case. For the larger displacement case (high nonlinearities), the two fundamental frequencies are much higher than the linear case - a result that is expected from a system with hard springs. Also, four other harmonics appeared in the identification output. Tables 10 and 11 summarize the identified quasi-linear modal parameters for the system. Table 11 shows that for the large amplitude case, the first two modes have the largest contributions to the responses, also indicating that these two modes are the fundamental modes.

For better understanding of the results in Table 11, the linear modes of the system would have resulted in modal contribution vectors of {0.2 0.2} and {0.1 -0.1} for the small amplitude case and {2.0 2.0} and {1.0 -1.0} for the larger amplitude case. For the nonlinear identified modal vectors, the amplitudes of the fundamental modes did not change much from the linear amplitudes, but large changes in the phase angles occurred.

Concluding Remarks

The time-domain, linear modal identification technique is found to be useful for the quasi-linear modal identification of nonlinear dynamic system. Such approach can be used to detect nonlinearities, and their types, in structures by performing the identification at different levels of response and study the changes in the identified modal parameters.

References

1. Klosterman, A. and Zimmerman, R., "Modal Survey Activity Via Frequency Response Functions." SAE Paper 751068, 1975.

2. Brown, D. L., Allemang, R. J., Zimmerman, R. and Mergeay, M., "Parameter Estimation Techniques for Modal Analysis." SAE Paper 790221, 1979.

3. Ibrahim, S. R. and Mikulcik, E. C., "The Experimental Determination of Vibration Parameters from Time Responses." Shock and Vibrations Bulletin, no. 46, part 5, August 1976, pp. 187-196.

4. Ibrahim, S. R. and Mikulcik, E. C., "A Method for the Direct Identification of Vibration Parameters from the Free Response." Shock and Vibration Bulletin, no. 47, part 4, September 1977, pp. 183-198.

5. Ibrahim, S. R., "Random Decrement Technique for Modal Identification of Structures." J. Spacecraft and Rockets, vol. 14, no. 11, November 1977, pp. 696-700.

6. Ibrahim, S. R., "Modal Confidence Factor in Vibration Testing." J. Spacecraft and Rockets, vol. 15, no. 5, September 1978, pp. 313-316.

7. Ibrahim, S. R., "Application of Random Time Domain Analysis to Dynamic Flight Measurements." Shock and Vibration Bulletin, no. 49, part 2, September 1979, pp. 165-170.

8. Hanks, B. R., Miserentino, R., Ibrahim, S. R., Lee, S. H., and Wada, B. K., "Comparison of Modal Test Methods on the Voyager Payload." SAE Paper 781044, November 1978.

9. Ibrahim, S. R. and Mikulcik, E. C., "Time Domain Identification of Standing Wave Parameters in Gas Piping Systems." J. Sound and Vibration, vol. 60, part 1, 1978, pp. 21-31.

10. Pappa, R. S. and Ibrahim, S. R., "A Parametric Study of the Ibrahim Time Domain Modal Identification Algorithm." Shock and Vibration Bulletin, no. 51, June 1981.

11. Ibrahim, S. R. and Pappa, R. S., "Large Modal Survey Testing Using the Ibrahim Time Domain (ITD) Identification Technique." AIAA Paper 81-0528-CP, April 1981.

12. Beliveau, J. G., "Identification of Viscous Damping in Structures from Model Information." J. Appl. Mech., ASME 43, pp. 335-339 (1976).

13. Berman, A., "System Identification of a Complex Structure." AIAA/ASME/SAE 16th Structures, Structural Dynamics, and Materials Conference, Denver, Colorado, May 27-29, 1975.

14. Collins, J. D., et al., "Statistical Identification of Structures." AIAA Journal, vol. 12, pp. 185-190, 1970.

15. Distefano, N. and Todeschini, R., "Modeling, Identification and Prediction of a Class of Nonlinear Viscoelastic Materials." International Journal of Solid and Structures (part 1), vol. 9, pp. 805-818, 1973.

16. Udwadia, F. E. and Shah, P. C., "Identification of Structures Through Records Obtained During Strong Earthquake Ground Motion." ASME, Journal of Engineering for Industry, vol. 98, no. 4, pp. 1347-1362, 1976.

17. Masri, S. F. and Caughey, T. K., "A Nonparametric Identification Technique for Non-Linear Dynamic Problems." J. Appl. Mech., ASME 46, pp. 433-447, 1979.

18. Udwadia, F. E. and Kuo, C. P., "Non-Parametric Identification of a Class of Non-Linear Close-Coupled Dynamic Systems." Earthquake Engn. and Structural Dynamics, vol. 9, pp. 385-409, 1981.

19. Yang, Y. and Ibrahim, S. R., "A Nonparametric Identification Technique for a Variety of Discrete Nonlinear Systems." To be presented at the 1982 ASME Winter Annual Meeting, Phoenix, Arizona, November 1982. Also under publication in ASME Transaction.

20. Holehouse, I., "Sonic Fatigue Design Techniques for Advanced Composite Aircraft Structures." AFWAL-TR-80-3019, Flight Dynamics Laboratory, Wright Aeronautical Laboratories, WPAFB, Ohio, April 1980.

21. Mei, C., "Large Amplitude Response of Complex Structures Due to High Intensity Noise." AFFDL-TR-79-3028, Flight Dynamics Laboratory, Wright Aeronautical Laboratories, WPAFB, Ohio, April 1979.

22. Mei, C., "Response of Nonlinear Structural Panels Subjected to High Intensity Noise." AFWAL-TR-80-3018, WPAFB, Ohio, March 1980.

23. Mei, C. and Wentz, K. R., "Large Amplitude Random Response Angle-Ply Laminated Composite Plates." Proceedings AIAA Dynamics Specialists Conference, Atlanta, Georgia, April 1981.

24. Chu, H. N and Herrmann, G., "Influence of Large Amplitudes on Free Flexural Vibrations of Rectangular Elastic Plates." J. Applied Mechanics, vol. 23, December 1956, pp. 532-540.

25. Mei, C. and Wentz, K. R., "Analytical and Experimental Nonlinear Response of Rectangular Panels to Acoustic Excitation." Proceedings of AIAA SDM Conference, New Orleans, May 1982, pp. 514-520.

26. White, R. G., "Comparison of the Statistical Properties of the Responses of Aluminum and CFRP Plates to Acoustic Excitation." J. Composites, October 1978, pp. 251-258.

APPENDIX A

Background - ITD Modal Identification Technique

The ITD modal identification technique is based on the fact that a vector of free-decay responses of $2m$ measurements, and or pseudo measurements, and containing contributions from p modes can be expressed as:

$$\{\phi(t)\} = \sum_{i=1}^{2p} \{\psi_i\} e^{\lambda_i t} \tag{A1}$$

To allow using an identification mathematical model with an m degrees of freedom, where $m >> p$, the vector $\{\phi(t)\}$ is arbitrarily selected as:

$$\{\phi(t)\} = \begin{Bmatrix} \{x(t)\} \\ \{x(t + \Delta t_2)\} \\ \{x(t + 2\Delta t_2)\} \\ \vdots \\ \{x(t + \Delta t_3)\} \\ \{x(t + \Delta t_2 + \Delta t_3)\} \\ \{x(t + 2\Delta t_2 + \Delta t_3)\} \\ \vdots \end{Bmatrix} \tag{A2}$$

where $\{x(t)\}$ is the measured response vector. Equation (1) can be written for the same responses delayed Δt_1 in time as:

$$\hat{\phi}(t)\} = \{\phi(t + \Delta t_1)\} = \sum_{i=1}^{2p} \{\psi_i\} e^{\lambda_i \Delta t_1} \cdot e^{\lambda_i t} \tag{A3}$$

By repeating these measurements vectors $\{\phi\}$ and $\{\hat{\phi}\}$ for $2r$ times where $r > m$, to form the two $2m \times 2r$ response matrices $[\phi]$ and $[\hat{\phi}]$, and computing a matrix $[A]$ where:

$$[A]\,[\phi][\phi]^T = [\hat{\phi}][\phi]^T \tag{A4}$$

the modal parameters of the system under consideration can be determined from the eigenvalue problem.

$$[A]\{\psi_i\} = e^{\lambda_i \Delta t_1} \{\psi_i\} \tag{A5}$$

For complete details on ITD please refer to references 3-11.

Table 1 Ratios to fundamental of identified frequencies for undamped simple pendulum

Mode no. (N)	$\theta_o = 30^o$		$\theta_o = 60^o$		$\theta_o = 90^o$		$\theta_o = 120^o$	
	f_N	f_N/f_1	f_N	f_N/f_1	f_N	f_N/f_1	f_N	f_N/f_1
1	0.9829	1.000	0.9318	1.000	0.8472	1.000	0.7284	1.000
2	2.9846	3.037	2.7949	2.999	2.5417	3.000	2.1854	2.999
3	4.9162	5.002	4.6747	5.017	4.2360	5.000	3.6415	4.999
4	-	-	6.5269	7.005	5.9334	7.004	5.0987	7.000
5	-	-	-	-	-	-	6 5681	9.017
6	-	-	-	-	-	-	-	-

Table 2 Ratios to fundamental of identified frequencies for undamped spring-mass system

Mode no. (N)	$y_o = 0.5$		$y_o = 1.0$		$y_o = 1.5$		$y_o = 2.0$	
	f_N	f_N/f_1	f_N	f_N/f_1	f_N	f_N/f_1	f_N	f_N/f_1
1	1.1708	1.000	1.5691	1.000	2.0651	1.000	2.6032	1.000
2	3.5124	3.000	4.7073	3.000	6.1953	3.000	7.8116	3.001
3	5.8529	4.999	7.8456	5.000	10.3256	5.000	13.0204	5.002
4	-	-	10.9841	7.000	14.4605	7.002	18.2261	7.001
5	-	-	-	-	-	-	23.4437	9.006
6	-	-	-	-	-	-	-	-

Table 3 Identified damping factors for undamped simple pendulum

θ_o	Damping factors* ζ_i				
	1	2	3	4	5
30	0.00000	0.00065	0.00002	-	-
60	0.00001	0 00000	0.00050	-0.00096	-
90	0.00006	0.00001	0.00000	0.00122	-
120	0.00000	0.00002	0.00030	0.00866	0.00593

*All theoretical damping factors were equal to zero.

*The listed values were obtained from different identification runs for optimum damping identification.

Table 4 Amplitudes of identified harmonics for undamped simple pendulum

Initial amplitude θ_o (rad.)		Identified $\theta_o = \theta_1 + \theta_3 + \theta_5 + \theta_7 + \theta_9$				
Theory	Identified	θ_1	θ_3	θ_5	θ_7	θ_9
0.5236	0.5236	0.5244×10^{0}	-0.7474×10^{-3}	-0.1080×10^{-4}	-	-
1.0472	1.0274	0.1054×10^{1}	-0.6429×10^{-2}	0.6942×10^{-4}	-0.2919×10^{-6}	-
1.5708	1.5708	0.1594×10^{1}	-0.2396×10^{-1}	0.6300×10^{-3}	-0.1930×10^{-4}	-
2.0944	2.0944	0.2158×10^{1}	-0.6697×10^{-1}	0.3449×10^{-2}	-0.2058×10^{-3}	0.1156×10^{-4}

Table 5 Amplitudes of identified harmonics for undamped spring-mass system

Initial amplitude y_o		Identified $y_o = y_1 + y_3 + y_5 + y_9$				
Theory	Identified	y_1	y_3	y_5	y_7	y_9
0.5000	0.5000	0.4942×10^{0}	0.5697×10^{-2}	0.6270×10^{-4}	-	-
1.0000	1.0000	0.9741×10^{0}	0.2529×10^{-1}	0.6399×10^{-3}	0.1577×10^{-4}	-
1.5000	1.5000	0.1449×10^{1}	0.4913×10^{-1}	0.1617×10^{-2}	0.6205×10^{-4}	-
2.0000	2.0005	0.1924×10^{1}	0.7323×10^{-1}	0.29871×10^{-2}	-0.3066×10^{-3}	0.7986×10^{-3}

Table 6 Theoretical and identified fundamental frequency for undamped simple pendulum

θ_o^o	f_1 (Hz)	
	Theory	Identified
30	0.9829	0.9829
60	0.9318	0.9318
90	0.8472	0.8472
120	0.7284	0.7284

Table 7 Theoretical and identified fundamental frequency for undamped spring-mass system

y_o	f_1 (Hz)	
	Theory	Identified
0.5	1.1708	1.1711
1.0	1.5691	1.5693
1.5	2.0651	2.0650
2.0	2.6032	2.6042

Table 8 Identified frequencies and damping factors for simple pendulum

θ_o^o	f (Hz)	ζ (%)
30	0.9906	0.93
	2.9186	3.50
60	0.9398	1.15
	2.7712	2.89
90	0.8678	1.30
	2.5698	2.60
120	0.7714	1.89
	2.2955	4.67

Table 9 Identified frequencies and damping factors for spring-mass system

y_o	f (Hz)	ζ (%)
0.5	1.1542	0.76
	3.4240	2.11
1.0	1.5242	0.49
	4.5873	1.98
1.5	1.9935	0.36
	6.0214	1.89
2.0	2.5064	0.33
	7.5700	1.58

Table 10 Identified frequencies and damping factors for the two-degree-of-freedom system*

y_{01}, y_{02}	Mode no.	f (Hz)	ζ (%)
0.3, 0.1	1	1.0185	0.95
	2	2.0359	0.99
3.0, 1.0	1	2.2783	1.40
	2	4.0214	1.34
	3	5.7955	1.07
	4	8.5792	0.65
	5	10.4157	1.47
	6	12.1253	1.60

*Theoretical linear frequencies are 1.0 and 2.0 Hz and damping factors are 1.0% for the two modes.

Table 11 Identified quasi-linear mode shapes for the two-degree-of-freedom system

Case (y_{01}, y_{02})	Station	Mode no. 1	2	3	4	5	6
0.3, 0.1	1 - Ampl.	0.1992	0.10156				
	Pha.^{o}	0.21	0.23				
	2 - Ampl.	0.1995	0.1014				
	Pha.^{o}	0.25	178.84				
3.0, 1.0	1 - Ampl.	2.0408	1.7412	0.0643	0.1191	0.0455	0.0578
	Pha.^{o}	-8.10	-29.30	-106.00	-71.94	-94.91	-126.47
	2 - Ampl.	1.9648	1.8263	0.0828	0.1121	0.0333	0.0547
	Pha.^{o}	-11.45	99.10	-154.79	-96.12	-35.31	78.94

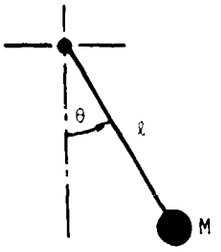
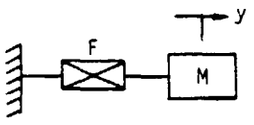


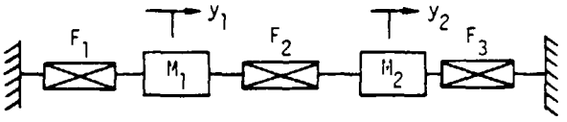
Fig. 1 Simple pendulum.



$m = 1$

$F = 4\pi^2(y + 2y^3) + 0.04\pi\dot{y}$

Fig. 2 Single-mass-spring system.



$M_1 = M_2 = 1$

$F_1 = F_3 = 4\pi^2(z + z^3) + 0.04\pi\dot{z}$

$F_2 = 6\pi^2(z + 0.5\,z^3) + 0.06\pi\dot{z}$

(z is displacement of restoring force element)

Fig. 3 Two-degree-of-freedom system.

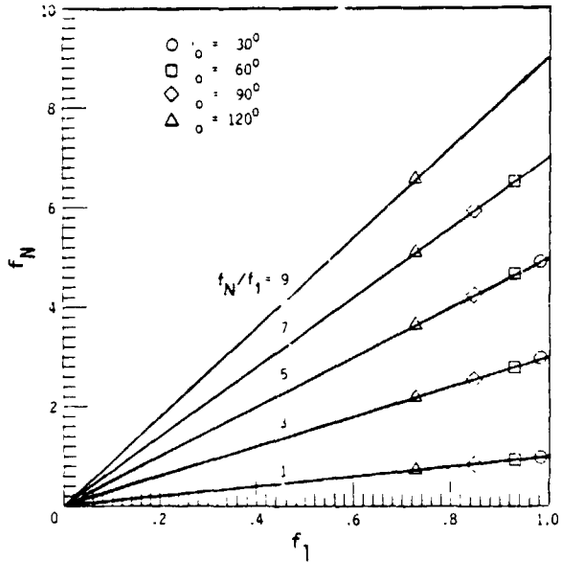


Fig. 4 Identified harmonics versus fundamental frequencies of simple pendulum.

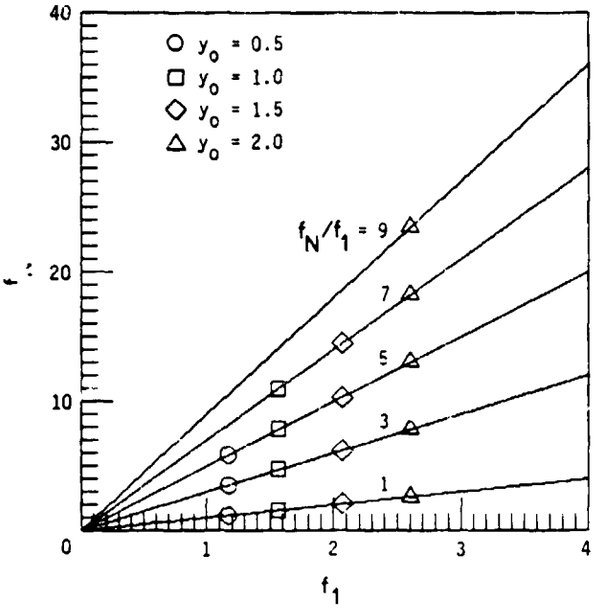


Fig. 5 Identified harmonics versus fundamental frequency for spring-mass system.

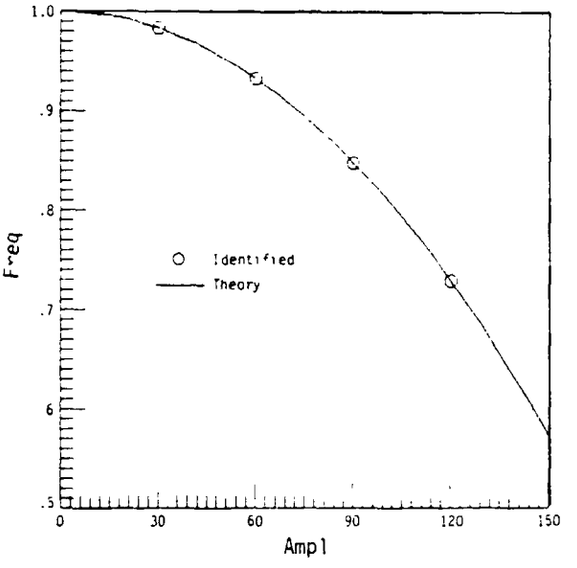


Fig. 6 Theoretical and identified fundamental frequency of simple pendulum.

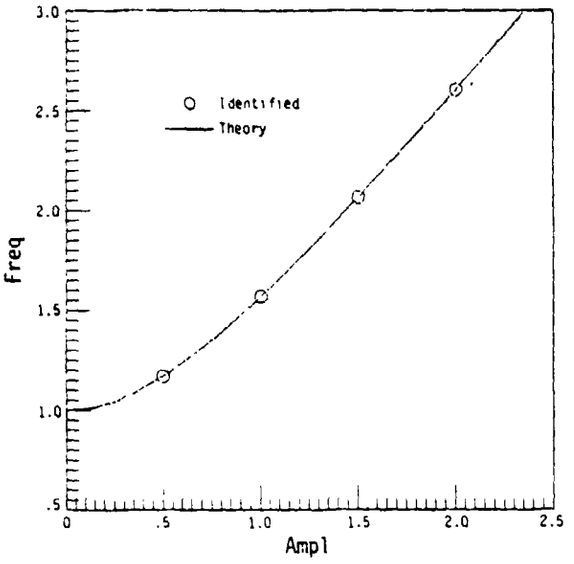


Fig. 7 Theoretical and identified fundamental frequency of spring-mass system.

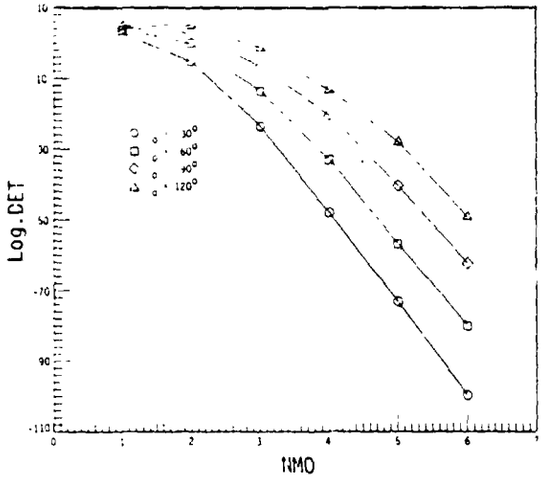


Fig. 8 Determinant versus DOF of identification model for simple pendulum.

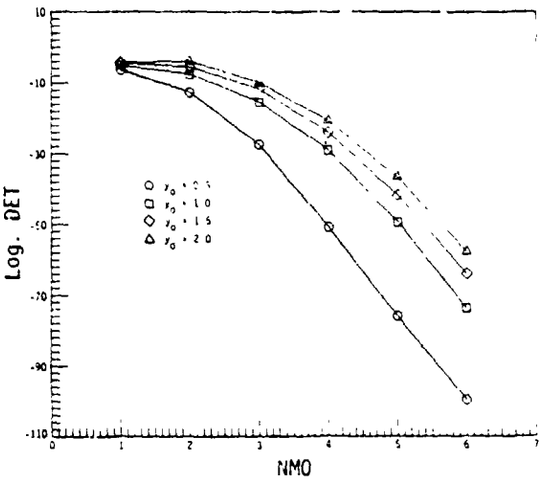


Fig. 9 Determinants versus DOF of identification model for spring-mass system.